4

TRIANGLES

2017B   21. In $\triangle ABC$, $AB = 6$, $AC = 8$, $BC = 10$, and D is the midpoint of $\overline{BC}$. What is the sum of the radii of the circles inscribed in $\triangle ADB$ and $\triangle ADC$?

(A) $\sqrt{5}$ (B) $\dfrac{11}{4}$ (C) $2\sqrt{2}$ (D) $\dfrac{17}{6}$ (E) 3

2002B 22. Let $\triangle XOY$ be a right-angled triangle with $m\angle XOY = 90°$. Let M and N be the midpoints of legs OX and OY, respectively. Given that $XN = 19$ and $YM = 22$, find XY.

(A) 24   (B) 26   (C) 28   (D) 30   (E) 32

2004B 22. A triangle with sides of 5, 12, and 13 has both an inscribed and a circumscribed circle. What is the distance between the centers of those circles?

(A) $\dfrac{3\sqrt{5}}{2}$ (B) $\dfrac{7}{2}$ (C) $\sqrt{15}$ (D) $\dfrac{\sqrt{65}}{2}$ (E) $\dfrac{9}{2}$

2017A 22. Sides $\overline{AB}$ and $\overline{AC}$ of equilateral triangle ABC are tangent to a circle at points B and C, respectively. What fraction of the area of $\triangle ABC$ lies outside the circle?

(A) $\dfrac{4\sqrt{3}\pi}{27} - \dfrac{1}{3}$ (B) $\dfrac{\sqrt{3}}{2} - \dfrac{\pi}{8}$ (C) $\dfrac{1}{2}$ (D) $\sqrt{3} - \dfrac{2\sqrt{3}\pi}{9}$

(E) $\dfrac{4}{3} - \dfrac{4\sqrt{3}\pi}{27}$

2009A

23. Convex quadrilateral $ABCD$ has $AB = 9$ and $CD = 12$. Diagonals $\overline{AC}$ and $\overline{BD}$ intersect at E, $AC = 14$, and $\triangle AED$ and $\triangle BEC$ have equal areas. What is AE?

(A) $\dfrac{9}{2}$ (B) $\dfrac{50}{11}$ (C) $\dfrac{21}{4}$ (D) $\dfrac{17}{3}$ (E) 6

2013A

23. In $\triangle ABC$, $AB = 86$, and $AC = 97$. A circle with center A and radius AB intersects $\overline{BC}$ at points B and X. Moreover $\overline{BX}$ and $\overline{CX}$ have integer lengths. What is BC?

(A) 11   (B) 28   (C) 33   (D) 61   (E) 72

2013B

23. In triangle ABC, $AB = 13$, $BC = 14$, and $CA = 15$. Distinct points D, E, and F lie on segments $\overline{BC}$, $\overline{CA}$, and $\overline{DE}$, respectively, such that $\overline{AD} \perp \overline{BC}$, $\overline{DE} \perp \overline{AC}$, and $\overline{AF} \perp \overline{BF}$. The length of segment $\overline{DF}$ can be written as $\dfrac{m}{n}$, where m and n are relatively prime positive integers. What is $m + n$?

(A) 18   (B) 21   (C) 24   (D) 27   (E) 30

2017A

23. How many triangles with positive area have all their vertices at points (i, j) in the coordinate plane, where i and j are integers between 1 and 5, inclusive?

(A) 2128   (B) 2148   (C) 2160   (D) 2200   (E) 2300

2004B

24. In $\triangle ABC$ we have $AB = 7$, $AC = 8$, and $BC = 9$. Point D is on the circumscribed circle of the triangle so that $\overline{AD}$ bisects $\angle BAC$. What is the value of AD/CD?

(A) $\dfrac{9}{8}$ (B) $\dfrac{5}{3}$ (C) 2   (D) $\dfrac{17}{7}$ (E) $\dfrac{5}{2}$

2017B

24. The vertices of an equilateral triangle lie on the hyperbola $xy = 1$, and a vertex of this hyperbola is the centroid of the triangle. What is the square of the area of the triangle?

(A) 48   (B) 60   (C) 108   (D) 120   (E) 169

2018A 24. Triangle ABC with $AB = 50$ and $AC = 10$ has area 120. Let D be the midpoint of $\overline{AB}$, and let E be the midpoint of $\overline{AC}$. The angle bisector of $\angle BAC$ intersects $\overline{DE}$ and $\overline{BC}$ at F and G, respectively. What is the area of quadrilateral $FDBG$?

(A) 60   (B) 65   (C) 70   (D) 75   (E) 80