8

COMBINATIONS

2003A

21. Pat is to select six cookies from a tray containing only chocolate chip, oatmeal, and peanut butter cookies. There are at least six of each of these three kinds of cookies on the tray. How many different assortments of six cookies can be selected?

(A) 22 **(B)** 25 **(C)** 27 **(D)** 28 **(E)** 729

2006A

21. How many four-digit positive integers have at least one digit that is a 2 or a 3?

(A) 2439 **(B)** 4096 **(C)** 4903 **(D)** 4904 **(E)** 5416

2008B

21. Ten chairs are evenly spaced around a round table and numbered clockwise from 1 through 10. Five married couples are to sit in the chairs with men and women alternating, and no one is to sit either next to or directly across from his or her spouse. How many seating arrangements are possible?

(A) 240 **(B)** 360 **(C)** 480 **(D)** 540 **(E)** 720

2013A

21. A group of 12 pirates agree to divide a treasure chest of gold coins among themselves as follows. The k^{th} pirate to take a share takes $\frac{k}{12}$ of the coins that remain in the chest. The number of coins initially in the chest is the smallest number for which this arrangement will allow each pirate to receive a positive whole number of coins. How many coins does the 12^{th} pirate receive?

(A) 720 **(B)** 1296 **(C)** 1728 **(D)** 1925 **(E)** 3850

2010A

22. Eight points are chosen on a circle, and chords are drawn connecting every pair of points. No three chords intersect in a single point inside the circle. How many triangles with all three vertices in the interior of the circle are created?

(A) 28 **(B)** 56 **(C)** 70 **(D)** 84 **(E)** 140

2010B

22. Seven distinct pieces of candy are to be distributed among three bags. The red bag and the blue bag must each receive at least one piece of candy; the white bag may remain empty. How many arrangements are possible?

(A) 1930 **(B)** 1931 **(C)** 1932 **(D)** 1933 **(E)** 1934

2011A

22. Each vertex of convex pentagon $ABCDE$ is to be assigned a color. There are 6 colors to choose from, and the ends of each diagonal must have different colors. How many different colorings are possible?

(A) 2520 **(B)** 2880 **(C)** 3120 **(D)** 3250 **(E)** 3750

2012B

22. Let $(a_1, a_2, \ldots, a_{10})$ be a list of the first 10 positive integers such that for each $2 \le i \le 10$ either $a_i + 1$ or $a_i - 1$ or both appear somewhere before a_i in the list. How many such lists are there?

(A) 120 **(B)** 512 **(C)** 1024 **(D)** 181,440 **(E)** 362,880

2016B

22. A set of teams held a round-robin tournament in which every team played every other team exactly once. Every team won 10 games and lost 10 games; there were no ties. How many sets of three teams $\{A, B, C\}$ were there in which A beat B, B beat C, and C beat A?

(A) 385 **(B)** 665 **(C)** 945 **(D)** 1140 **(E)** 1330

2008A

23. Two subsets of the set $S = \{a, b, c, d, e\}$ are to be chosen so that their union is S and their intersection contains exactly two elements. In how many ways can this be done, assuming that the order in which the subsets are chosen does not matter?

(A) 20 **(B)** 40 **(C)** 60 **(D)** 160 **(E)** 320

2008B 23. A rectangular floor measures a feet by b feet, where a and b are positive integers with $b > a$. An artist paints a rectangle on the floor with the sides of the rectangle parallel to the sides of the floor. The unpainted part of the floor forms a border of width 1 foot around the painted rectangle and occupies half the area of the entire floor. How many possibilities are there for the ordered pair (a, b)?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

2010B 23. The entries in a 3×3 array include all the digits from 1 through 9, arranged so that the entries in every row and column are in increasing order. How many such arrays are there?

(A) 18 **(B)** 24 **(C)** 36 **(D)** 42 **(E)** 60

2012A 23. Adam, Benin, Chiang, Deshawn, Esther, and Fiona have internet accounts. Some, but not all, of them are internet friends with each other, and none of them has an internet friend outside this group. Each of them has the same number of internet friends. In how many different ways can this happen?

(A) 60 **(B)** 170 **(C)** 290 **(D)** 320 **(E)** 660

2003A

24. Sally has five red cards numbered 1 through 5 and four blue cards numbered 3 through 6. She stacks the cards so that the colors alternate and so that the number on each red card divides evenly into the number on each neighboring blue card. What is the sum of the numbers on the middle three cards?

(A) 8 **(B)** 9 **(C)** 10 **(D)** 11 **(E)** 12

2012B

24. Amy, Beth, and Jo listen to four different songs and discuss which ones they like. No song is liked by all three. Furthermore, for each of the three pairs of the girls, there is at least one song liked by those two girls but disliked by the third. In how many different ways is this possible?

(A) 108 **(B)** 132 **(C)** 671 **(D)** 846 **(E)** 1105

2013A

24. Central High School is competing against Northern High School in a backgammon match. Each school has three players, and the contest rules require that each player play two games against each of the other school's players. The match takes place in six rounds, with three games played simultaneously in each round. In how many different ways can the match be scheduled?

(A) 540 **(B)** 600 **(C)** 720 **(D)** 810 **(E)** 900

2014B

24. The numbers 1, 2, 3, 4, 5 are to be arranged in a circle. An arrangement is *bad* if it is not true that for every n from 1 to 15 one can find a subset of the numbers that appear consecutively on the circle that sum to n. Arrangements that differ only by a rotation or a reflection are considered the same. How many different bad arrangements are there?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

2015A

24. For some positive integers p, quadrilateral $ABCD$ with positive integer side lengths has perimeter p, right angles at B and C, $AB = 2$, and $CD = AD$. How many different values of $p < 2015$ are possible?

(A) 30 **(B)** 31 **(C)** 61 **(D)** 62 **(E)** 63

2003A

25. Let n be a 5-digit number, and let q and r be the quotient and remainder, respectively, when n is divided by 100. For how many values of n is $q + r$ divisible by 11?

(A) 8180 **(B)** 8181 **(C)** 8182 **(D)** 9000 **(E)** 9090

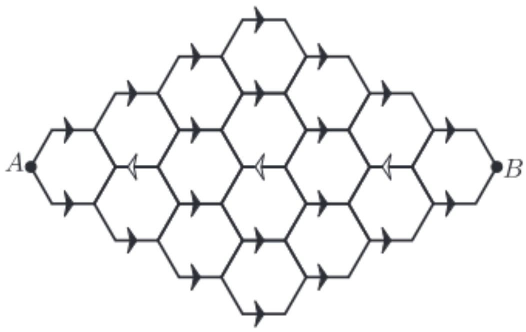
2005B

25. A subset B of the set of integers from 1 to 100, inclusive, has the property that no two elements of B sum to 125. What is the maximum possible number of elements in B?

(A) 50 **(B)** 51 **(C)** 62 **(D)** 65 **(E)** 68

2012B

25. A bug travels from A to B along the segments in the hexagonal lattice pictured below. The segments marked with an arrow can be traveled only in the direction of the arrow, and the bug never travels the same segment more than once. How many different paths are there?



(A) 2112 **(B)** 2304 **(C)** 2368 **(D)** 2384 **(E)** 2400

2013A

25. All 20 diagonals are drawn in a regular octagon. At how many distinct points in the interior of the octagon (not on the boundary) do two or more diagonals intersect?

(A) 49 **(B)** 65 **(C)** 70 **(D)** 96 **(E)** 128

2017A 25. How many integers between 100 and 999, inclusive, have the property that some permutation of its digits is a multiple of 11 between 100 and 999? For example, both 121 and 211 have this property.

(A) 226 **(B)** 243 **(C)** 270 **(D)** 469 **(E)** 486