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ARITHMETIC

2004B   6. Which of the following numbers is a perfect square?

(A) $98! \cdot 99!$ (B) $98! \cdot 100!$ (C) $99! \cdot 100!$ (D) $99! \cdot 101!$ (E) $100! \cdot 101!$

2003A
6. Define $x \heartsuit y$ to be $|x - y|$ for all real numbers x and y. Which of the following statements is **not** true?

(A) $x \heartsuit y = y \heartsuit x$ for all x and y

(B) $2(x \heartsuit y) = (2x) \heartsuit (2y)$ for all x and y (C) $x \heartsuit 0 = x$ for all x

(D) $x \heartsuit x = 0$ for all x (E) $x \heartsuit y > 0$ if $x \neq y$

2010A
6. For positive numbers x and y the operation $\spadesuit(x, y)$ is defined as

$$\spadesuit(x, y) = x - \frac{1}{y}.$$

What is $\spadesuit(2, \spadesuit(2, 2))$?

(A) $\frac{2}{3}$ (B) 1   (C) $\frac{4}{3}$ (D) $\frac{5}{3}$ (E) 2

2003B

7. The symbolism $\lfloor x \rfloor$ denotes the largest integer not exceeding x. For example, $\lfloor 3 \rfloor = 3$, and $\lfloor 9/2 \rfloor = 4$. Compute

$$\lfloor \sqrt{1} \rfloor + \lfloor \sqrt{2} \rfloor + \lfloor \sqrt{3} \rfloor + \cdots + \lfloor \sqrt{16} \rfloor.$$

(A) 35   (B) 38   (C) 40   (D) 42   (E) 136

2006B

7. Which of the following is equivalent to $\sqrt{\dfrac{x}{1 - \frac{x-1}{x}}}$ when $x < 0$?

(A) $-x$ (B) x (C) 1 (D) $\sqrt{\dfrac{x}{2}}$ (E) $x\sqrt{-1}$

2009B

7. By inserting parentheses, it is possible to give the expression

$$2 \times 3 + 4 \times 5$$

several values. How many different values can be obtained?

(A) 2   (B) 3   (C) 4   (D) 5   (E) 6

2013A

8. What is the value of

$$\frac{2^{2014} + 2^{2012}}{2^{2014} - 2^{2012}}?$$

(A) -1 (B) 1 (C) $\dfrac{5}{3}$ (D) 2013 (E) 2^{4024}

2008A

7. The fraction

$$\frac{\left(3^{2008}\right)^2 - \left(3^{2006}\right)^2}{\left(3^{2007}\right)^2 - \left(3^{2005}\right)^2}$$

simplifies to which of the following?

(A) 1 (B) $\dfrac{9}{4}$ (C) 3 (D) $\dfrac{9}{2}$ (E) 9

2015B 7. Consider the operation "minus the reciprocal of," defined by $a \diamond b = a - \frac{1}{b}$. What is $\left((1 \diamond 2) \diamond 3\right) - \left(1 \diamond (2 \diamond 3)\right)$?

(A) $-\dfrac{7}{30}$ (B) $-\dfrac{1}{6}$ (C) 0 (D) $\dfrac{1}{6}$ (E) $\dfrac{7}{30}$

2016B 8. What is the tens digit of $2015^{2016} - 2017$?

(A) 0 (B) 1 (C) 3 (D) 5 (E) 8

2014A 8. Which of the following numbers is a perfect square?

(A) $\dfrac{14!15!}{2}$ (B) $\dfrac{15!16!}{2}$ (C) $\dfrac{16!17!}{2}$ (D) $\dfrac{17!18!}{2}$ (E) $\dfrac{18!19!}{2}$

2003A 9. Simplify

$$\sqrt[3]{x\sqrt[3]{x\sqrt[3]{x\sqrt{x}}}}.$$

(A) $\sqrt{x}$ (B) $\sqrt[3]{x^2}$ (C) $\sqrt[27]{x^2}$ (D) $\sqrt[54]{x}$ (E) $\sqrt[81]{x^{80}}$